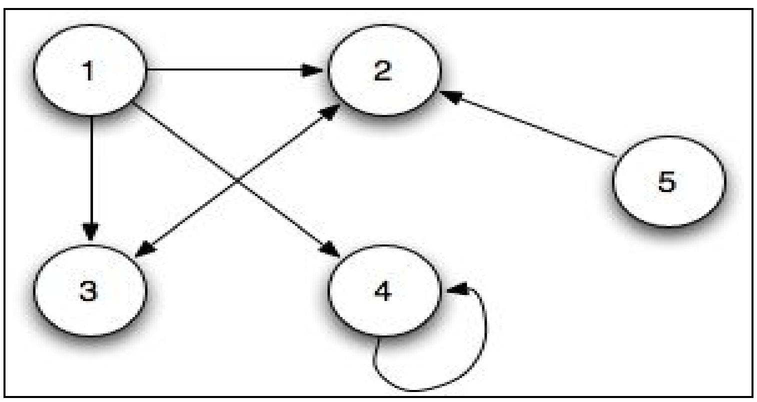
Name________________________
Date______________

# Markov Chains

A Markov chain is an aspect of Linear Algebra that businesses can use to estimate the distribution of a population over a period of time. Markov chains require a population matrix and a transition matrix, which are usually taken from graphs. But what is a graph?

The graphs that we will be working with are from Graph Theory. The graphs will be combinations of dots and lines that represent the movement of something from one place to another. An example of a graph is the following. Notice how each line coming from a point has an arrow indicating which direction it is moving.




Use the space below to draw a graph that has 3 points.

We can use graphs like those to help us organize our Markov chains. What we need to do is use information that is given to us to form 2 matrices: a population matrix and a transition matrix.

**Population Matrix:** A matrix that describes the distribution of a population at each point on our graph

**Transition Matrix**: A matrix that describes the movement of the population between points. The “action” matrix

To carry out our Markov chain, we need to use the dot product between the population matrix and the transition matrix. Let’s look at an example of a problem to get a better understanding.

Let’s say, for example, there are 2 coffee shops across the street from one another (Coffee A and Coffee B), and 500 people are split evenly between the 2 coffee shops. We want to know which of the shops will be doing better over time. We know that for each coffee shop, there are some people that return to the same shop the next morning and some people that go to the other shop the next morning.

With just this information, we can start to draw a graph to describe the problem. Use the space below to draw what you think the graph will look like. I’ll start you off with our 2 points.

Now we need to look for some more information on the problem. Let's say that 40% of the population from Shop A stays at shop A, while 60% of the population from shop A goes to shop B. Let's also say that 70% of the population of shop B stays at shop B, while 30% of the population from shop B goes to shop A. Modify your graph from above to include the following information

40% A➡A
60% A➡B
70% B➡B
30% B➡A

Also include that the population is split evenly at the start of day 1.


We can now use this directed graph to form both our population matrix and our transition matrix.

**Population Matrix**

We know that the population is split up evenly at the start, so create a 1X2 matrix that shows the population is split evenly between both shops

**Transition Matrix**

We want our transition matrix to account for every possible movement of the population, and we have 4 options that were mentioned earlier. We can create a 2X2 matrix where the top left entry represents the movement from A to A, the top right entry represents the movement from A to B, the bottom left entry represents the movement from B to A, and the bottom right entry represents the movement from B to B. Draw that matrix below for this given problem.

To carry out the Markov chain, we must now do the dot product between the population matrix and the transition matrix. Carry out the operation below.

This gives us the distribution of the population after the first day. What are some things you notice about the new population matrix?

Now use the new population matrix and do the dot product with that matrix and the transition matrix to get the population for day 3.

Which business do you feel will be more likely to succeed over time? What are some changes you would propose to the other business to help it out?